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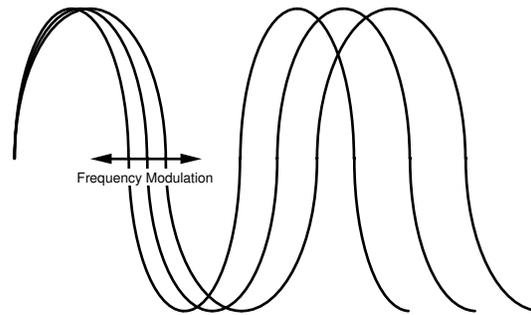
## [Assignment Questions](#)

## [For Further Research](#)



## 4.0 Frequency Modulation

There are three parameters in a sinewave carrier that may be varied or modulated to convey information: amplitude, frequency, and phase.



Notice that frequency modulation looks very much like phase modulation. They are in fact very similar, and many textbooks refer to them both as angle modulation.

<http://robotics.eecs.berkeley.edu/~sastry/ee20/modulation/modulation.htm>

↓

<http://www.radiodesign.com/radwrks.htm>

[Operating a Quadrature Modulator as an FSK Modulator by Micro Devices](#)

### 4.1 FM Theory

Recall that a general sinusoid is of the form:

$$e_c = \sin(\omega_c t + \theta)$$

Frequency modulation involves deviating a carrier frequency by some amount. If a sine wave was used to frequency modulate a carrier, the mathematical expression would be:

$$\omega_i = \omega_c + \Delta\omega \sin \omega_m t$$

where

$\omega_i$  = instantaneous frequency

$\omega_c$  = carrier frequency

$\Delta\omega$  = carrier deviation

$\omega_m$  = modulation frequency

This expression shows a signal varying sinusoidally about some average frequency. However, we cannot simply substitute expression in the general equation for a sinusoid. This is because the sine operator acts upon angles, not frequency. Therefore, we must define the instantaneous frequency in terms of angles.

It should be noted that the amplitude of the modulation signal governs the amount of carrier deviation, while the modulation frequency governs the rate of carrier deviation.

The term  $\omega$  is an angular velocity and it is related to frequency and angle by the following relationship:

$$\omega = 2\pi f = \frac{d\theta}{dt}$$

To find the angle, we must integrate  $\omega$  with respect to time, we obtain:

$$\int \omega dt = \theta$$

We can now find the instantaneous angle associated with an instantaneous frequency:

$$\begin{aligned} \theta &= \int \omega_i dt = \int (\omega_c + \Delta\omega \sin \omega_m t) dt \\ &= \omega_c t - \frac{\Delta\omega}{\omega_m} \cos \omega_m t = \omega_c t - \frac{\Delta f}{f_m} \cos \omega_m t \end{aligned}$$

This angle can now be substituted into the general carrier signal to define FM:

$$e_{fm} = \sin\left(\omega_c t - \frac{\Delta f}{f_m} \cos \omega_m t\right)$$

The FM modulation index is defined as the ratio of carrier deviation to modulation frequency:

$$m_{fm} = \frac{\Delta f}{f_m}$$

As a result, the FM equation is generally written as:

$$e_{fm} = \sin(\omega_c t - m_{fm} \cos \omega_m t)$$

This is a very complex expression and it is not readily apparent what the sidebands of this signal are like. The solution to this problem requires knowledge of Bessel's functions of the first kind and order  $p$ . In open form, it resembles:

$$J_p(x) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{x}{2}\right)^{2k+p}}{k!(k+p)!}$$

where

$J_p(x)$  = magnitude of frequency component  
 $p$  = side frequency number  
 $x$  = modulation index

As a point of interest, Bessel's functions are a solution to the following equation:

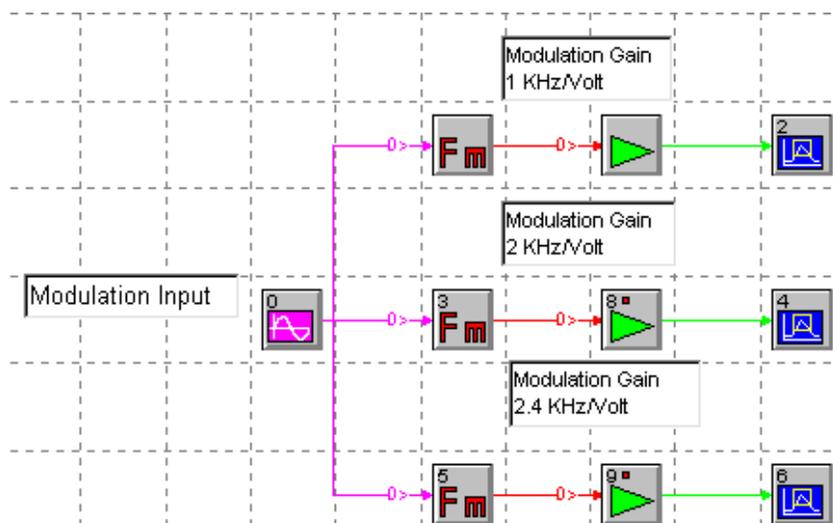
$$x^2 + \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - p^2) = 0$$

Bessel's functions occur in the theory of cylindrical and spherical waves, much like sine waves occur in the theory of plane waves.

It turns out that FM generates an infinite number of side frequencies. Each frequency is an integer multiple of the modulation signal. It should be noted that the amplitude of the higher order sided frequencies drops off quickly.

It is also interesting to note that the amplitude of the carrier signal is also a function of the modulation index. Under some conditions, the amplitude of the carrier frequency can actually go to zero. This does not mean that the signal disappears, but rather that all of the broadcast energy is redistributed to the side frequencies.

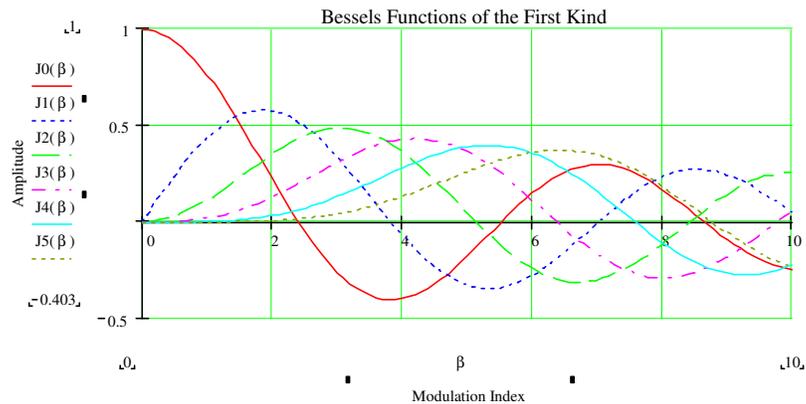
SystemView Model: FM Spectra vs. Modulation Index



[MathCAD Worksheet: Bessel's Functions](#)

[MathCAD Worksheet: FM](#)

A plot of the amplitudes of the carrier and first five side frequencies as a function of modulation index resembles:



The open form of the Bessel's function can be used to determine the magnitude of any spectral component.

[Frequency Modulation Movie Clip](#)

Example

Given a 5-volt peak sinewave, frequency modulated with a modulation index of  $m_f = 1$ , find the amplitude of the carrier and first side frequency.

Solution:

The Bessel coefficient for the carrier is:

$$J_0(1) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1}{2}\right)^{2k+0}}{k!(k+0)!} \approx 0.766$$

Therefore the carrier amplitude is  $5 \times 0.766 = 3.83V_{\text{peak}}$

The first side frequency coefficient is:

$$J_1(1) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1}{2}\right)^{2k+1}}{k!(k+1)!} \approx 0.4401$$

Therefore the amplitude is  $5 \times 0.4401 = 2.202V_{\text{peak}}$

FM generates an infinite number of side frequencies and therefore requires an infinite bandwidth. However, the side frequency amplitudes tend to drop off fairly quickly. If on those side frequencies with amplitudes  $<5\%$  of the unmodulated carrier are considered, the bandwidth may be approximated by Carson's rule:

$$B_{FM} \approx 2(m_{fm} + 1)f_m = 2(\Delta f + f_m)$$

Notice how the broadcast bandwidth changes as a function of the modulation frequency.

$f_m$ [KHz]	Bandwidth [KHz]
0.1	150
1	152
10	170

Because the bandwidth does not increase rapidly as the modulation frequency increases, FM is sometimes called a constant bandwidth system. And since the relatively low bandwidth modulation signal is spread out over a much larger transmission bandwidth, FM is also called spread spectrum.

In commercial broadcast applications, the deviation  $\Delta f$  is limited to 75 KHz and the maximum modulation frequency is limited to 15 KHz. The total broadcast bandwidth is therefore 180 KHz, but an additional 20 KHz is assigned as a guard band between stations.

The maximum deviation ratio is defined as:

$$M = \frac{\Delta F}{F_M} = \frac{75 \text{ KHz}}{15 \text{ KHz}} = 5$$

In narrow band applications such as radio dispatch systems, the amount of carrier deviation and modulation frequency are both significantly reduced.

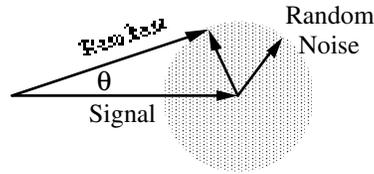
Because the amplitude of a frequency modulated signal remains constant, the power in the transmitted signal remains constant. As the modulation index is varied, the power gets redistributed among the side frequencies. With a modulation index of about 2.4, the frequency component associated with the unmodulated carrier disappears.

$$P_T = P_C (J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \dots))$$

$$J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \dots) = 1$$

### 4.1.1 Noise

In AM systems, noise easily distorts the transmitted signal, however, in FM systems, any added noise must create a frequency shift in order to be perceptible.



The maximum frequency deviation due to random noise occurs when the noise signal is at right angles to the resultant signal. In this worst case, the signal frequency has been deviated by:

$$\delta = \theta f_m$$

This may seem like a lot however, it is not.

#### Example

Calculate the signal to noise ratio of a frequency modulated signal if the noise burst is half the amplitude of the signal.

Solution:

The S/N ratio is  $1/0.5 = 2$ . This will generate a phase shift of:

$$\theta = \sin^{-1}\left(\frac{N}{S}\right) = \sin^{-1}\left(\frac{.5}{1}\right) = 30^\circ \text{ or } .52 \text{ radians}$$

Since the maximum modulation frequency for FM radio is 15 KHz, this amount of noise creates a frequency deviation of:

$$\delta = \theta f_m = .52 \times 15 \text{KHz} \approx 7.5 \text{KHz}$$

From the receiver point of view, the signal to noise ratio is the ratio of two deviations: the normal signal deviation, and the noise deviation.

$$\frac{S}{N} = \frac{75 \text{KHz}}{7.5 \text{KHz}} = 10$$

From this we observe that FM has a much better S/N than AM under the same conditions. This improvement in S/N ratio is due to spectral spreading, and helps to explain why the quality of FM radio is better than AM.

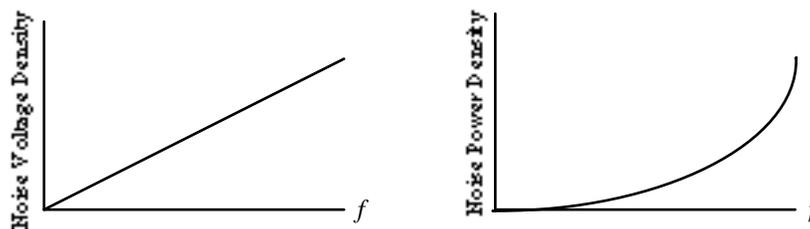
Another interesting effect associated with FM is the capture effect. In AM broadcast, it is possible to hear two stations simultaneously when they occupy the same frequency slot. This rarely happens in FM because the receiver will lock on to the stronger of the two FM carriers, and track its deviation, thus ignoring the other. This is generally an advantage however, in CDMA cellular radio systems, this phenomenon creates a problem.

4.1.1.1 Pre/De-emphasis

The fact that the carrier deviation in the presence of noise is given by:

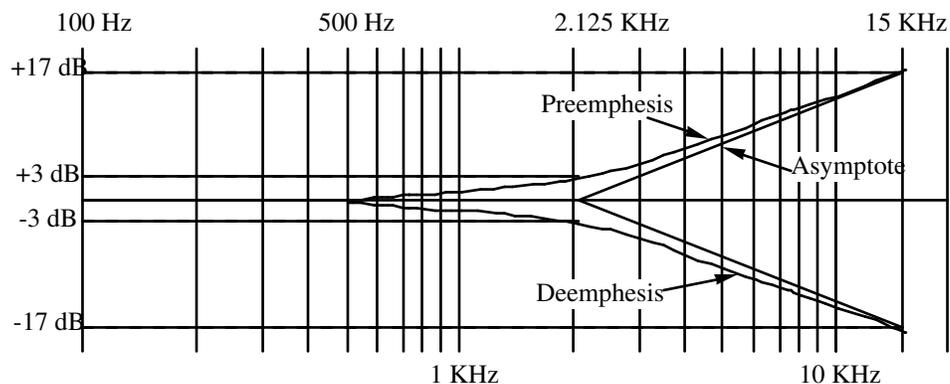
$$\delta = \theta f_m$$

implies that noise voltage density increases linearly with frequency. Since power increases as the square of voltage, we obtain the following noise characteristics:

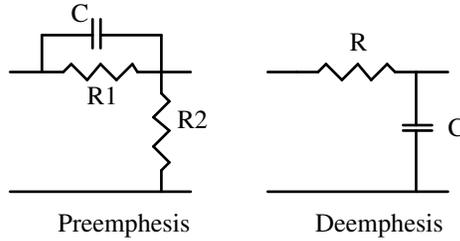


In order to keep a constant S/N ratio over the entire broadcast band, it is necessary to pre-emphasize or boost high frequency signals. This naturally requires performing the opposite function at the receiver, otherwise the signal would sound quite tinny.

The standard pre-emphasis and de-emphasis curves resemble:



In commercial FM broadcast, the pre-emphasis and de-emphasis circuits consist of a simple RC network. The RC time constant is 75 μSec, and the corner frequency is 2125 Hz.



The magnitude of the pre-emphasis response is defined by:

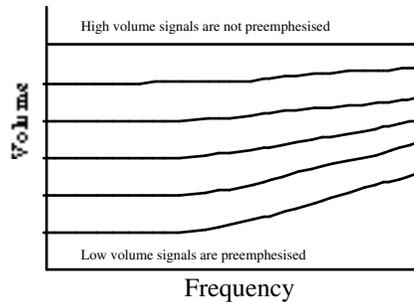
$$\left| \frac{V_o}{V_{in}} \right| = 20 \log \left[ \frac{R_2}{R_1 + R_2} \sqrt{1 + \left( \frac{f}{f_c} \right)^2} \right]$$

$$f_c = 2125 \text{ Hz}$$

4.1.1.2 Dolby

<http://www.dolby.com/>

In the Dolby system, pre-emphasis is a function of loudness. High volume, high frequency signals are not pre-emphasized. This prevents high fidelity music from pushing an FM station outside of its assigned frequency slot. This system is also used to reduce background hiss associated with magnetic tapes.

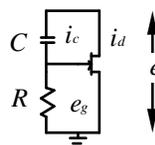


[Dolby B Compression and Expansion Curve](#)

4.2 Direct FM

4.2.1 Reactance Modulator

The reactance modulator is a voltage controlled capacitor and is used to vary an oscillator's frequency or phase. A simplified circuit resembles:



Since the gate does not draw an appreciable amount of current, applying Ohm's law in the RC branch results in:

$$e_g = i_C R$$

$$i_C = \frac{e}{R - jX_C}$$

$$\therefore e_g = \frac{e}{R - jX_C} R$$

The JFET drain current is given by:

$$i_d = g_m e_g = g_m \frac{e}{R - jX_C} R$$

where  $g_m$  is the transconductance.

The impedance as seen from the drain to ground is given by:

$$Z = \frac{e}{i_d} = e \frac{1}{g_m} \frac{R - jX_C}{e} \frac{1}{R} = \frac{1}{g_m} - j \frac{X_C}{g_m R}$$

Since transconductance is normally very large, the impedance reduces to:

$$Z \approx -j \frac{X_C}{g_m R} = \frac{-j}{2\pi f C g_m R}$$

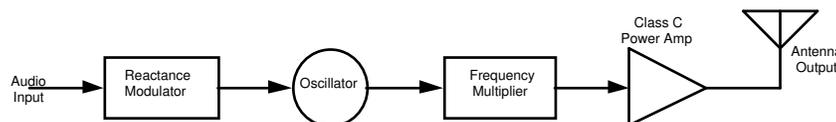
The term in the denominator can be thought of as an equivalent capacitance:

$$C_{eq} = C g_m R$$

Then

$$Z = \frac{-j}{2\pi f C_{eq}}$$

Since the equivalent capacitance is larger than the original capacitor, we have created a capacitance amplifier. Because the value of this capacitance is a function of applied voltage, we actually have a voltage controlled capacitor. This device can be used to control an oscillator frequency, thus producing FM.



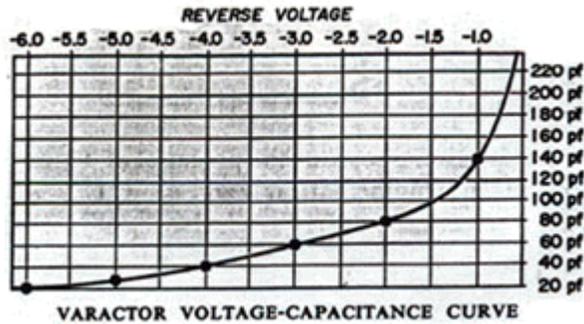
4.2.1.1 Varactor Diodes

<http://www.americamicrosemi.com/tutorials/varactor.htm>

The capacitance of a varactor diode is a function of its' reverse bias voltage.

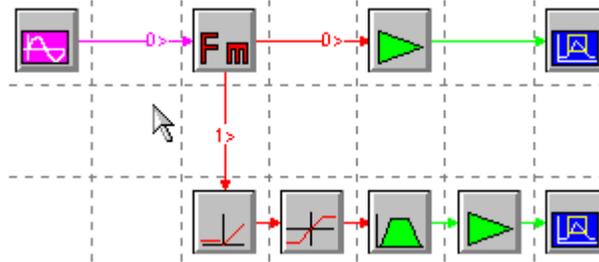
$$C_d = \frac{C_0}{\sqrt{1 + 2|V_R|}}$$

Where  $C_0$  is the diode capacitance at zero bias, and  $V_R$  is the reverse bias voltage. A typical response is:

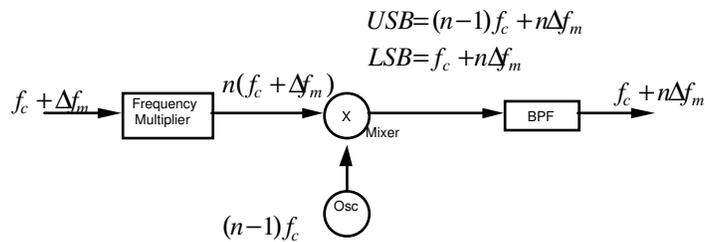


4.2.2 Frequency Deviation Amplifier

SystemView Model: Frequency Deviation Amplifier



Most modulators can only deviate a carrier a small amount. In order to increase the amount of frequency deviation, a combination of frequency multipliers and down converter is used.

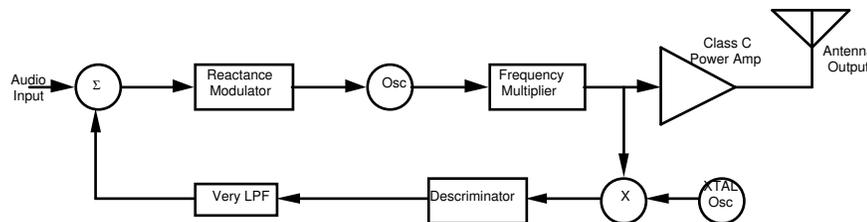


A frequency multiplier is not quite the same thing as a mixer. A mixer performs time domain multiplication and therefore shifts a frequency band up or down. A frequency multiplier not only shifts a frequency band up, but also increases the band width or carrier deviation. This is because the multiplication is performed in the frequency domain.

A frequency multiplier can be made by passing an FM signal through a class C amplifier, and filtering off the harmonic band of interest. This type of device can only provide integer multiplication. Normally a string of doublers and triplers are used to increase the carrier deviation to the desired amount. A mixer can then be used to shift the signal to the correct carrier frequency.

### 4.2.3 Crosby Modulator

The Crosby modulator uses a frequency stabilized reactance modulator.



The performance of transistor circuits is often quite dependent on temperature if some intrinsic characteristic such as transconductance is used. Even small variation in the performance of these circuits would cause a drastic change in the transmitted signal, since the variations would be magnified by the multipliers and mixers. To prevent this, a crystal oscillator is used as a reference.

If the carrier signal deviates from its assigned position, a difference frequency is created. This difference frequency is converted to an error voltage and used to correct the reactance modulator, thus bringing the carrier signal back to its proper location. A very low pass filter is required in the feedback loop in order to prevent carrier deviations associated with the modulation signal from correcting the modulator.

## 4.3 Indirect FM

Because crystal oscillators are so stable, it is desirable to use them in modulator circuits. However, their extreme stability makes it difficult to modulate their frequency.

Fortunately, it is possible to vary the phase of a crystal oscillator. However, in order to use this as an FM source, the relationship between frequency and phase needs to be reexamined.

Frequency is the rate of change of angle, its first derivative:

$$\omega = \frac{d}{dt} \phi$$

The instantaneous phase angle is comprised of two components, the number of times the signal has gone through its cycle, and its starting point or offset:

$$\phi(t) = \underbrace{\omega_c t}_{\text{rotating angle}} + \underbrace{\theta}_{\text{offset angle}}$$

The instantaneous frequency is therefore:

$$\omega_i = \frac{d}{dt} \phi(t) = \frac{d}{dt} [\omega_c t + \theta] = \omega_c + \frac{d}{dt} \theta$$

From this we observe that the instantaneous frequency of a signal is its unmodulated frequency plus a change. This is equivalent to frequency modulation. Therefore we may write:

$$\omega_c + \frac{d}{dt} \theta = \omega_c + \omega_{eq}$$

$$\omega_{eq} = \frac{d}{dt} \theta$$

$$f_{eq} = \frac{1}{2\pi} \frac{d}{dt} \theta$$

This means that the output of a phase modulator is proportional to the equivalent frequency modulation.

If the angle is proportional to the amplitude of a modulation signal  $\theta = k e_m$ , Then:

$$f_{eq} = \frac{1}{2\pi} \frac{d}{dt} k e_m$$

and by integrating the modulation signal prior to modulation, we obtain:

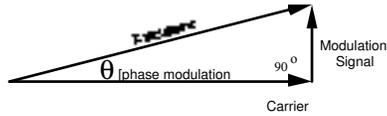
$$f_{eq} = \frac{1}{2\pi} \frac{d}{dt} \int k e_m dt = \frac{k}{2\pi} e_m$$

This means that the equivalent frequency modulation is directly proportional to the amplitude of a phase modulation signal if the modulation signal is integrated first.

This indirect modulation scheme is the heart of the Armstrong modulator.

### 4.3.1 Armstrong Modulator

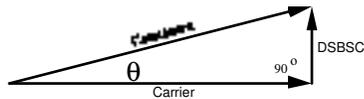
The phase of any signal can be easily varied by means of a quadrature modulator. A quadrature modulator has once oscillator, but two outputs. These outputs are at 90° to each other. By varying the amplitude of one or both of these outputs, and then adding them together, we obtain phase modulation. This technique is used in QPSK modems and broadcast video chroma signals.



$$\theta = \tan^{-1} \left( \frac{e_m}{e_c} \right)$$

For small angle changes [  $\theta \leq 10^\circ$  ], the tangent of an angle is approximately equal to the angle itself  $\tan \theta \approx \theta$ . This means that the resultant angle is proportional to the modulation signal amplitude.

The Armstrong modulator uses a DSBSC modulation signal as the modulation source.

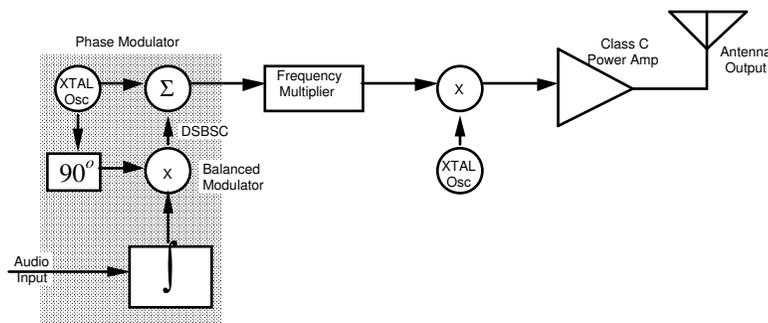


$$\theta = \tan^{-1} \left( \frac{e_{DSBSC}}{e_c} \right)$$

For small angle changes the phase modulation is proportional to the amplitude of the modulation signal.  $\theta \approx \frac{e_{DSBSC}}{e_c}$ .

By integrating the modulation signal, the phase modulation is converted to an equivalent frequency modulation.

The block diagram of an Armstrong modulator resembles:

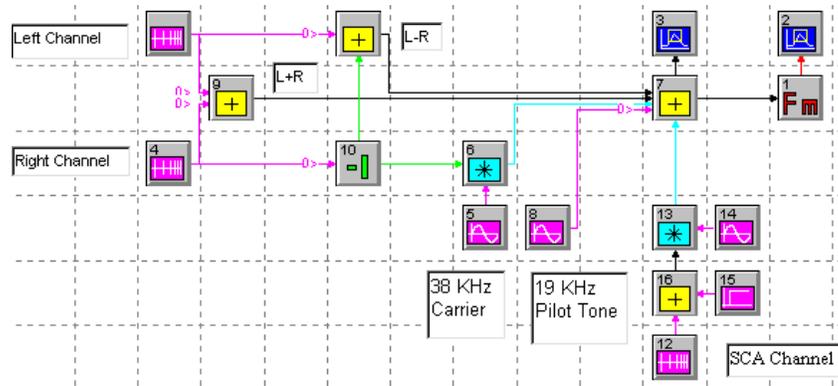


The carrier source is a low frequency [100 KHz] crystal oscillator. This provides a stable frequency and is within the operating region of a reactance modulator.

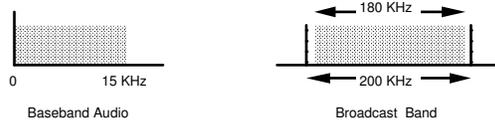
The highest permitted FM broadcast power in the US is 100 kilowatts. However, some stations built prior to the present regulations have higher power levels.

### 4.4 FM Stereo

#### SystemView Model: FM Stereo

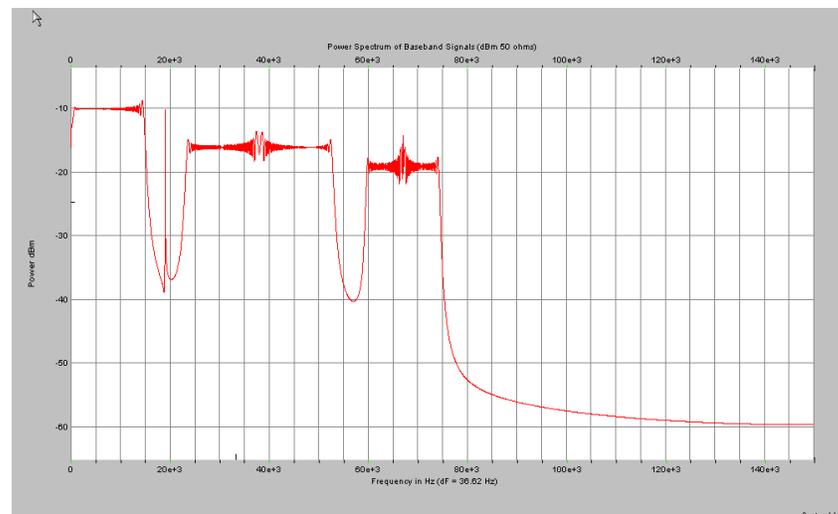


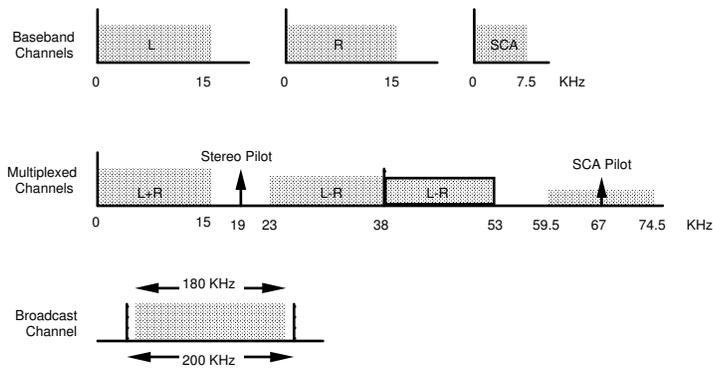
The baseband signal in a standard FM channel is 15 KHz, and the broadcast band is 180 KHz plus 20 KHz of guard band.



Stereo requires that there be two base band signals. Many radio stations also broadcast a 3rd channel which is used for background music in various business locations. These baseband signals are frequency multiplexed together and used to frequency modulate the FM carrier.

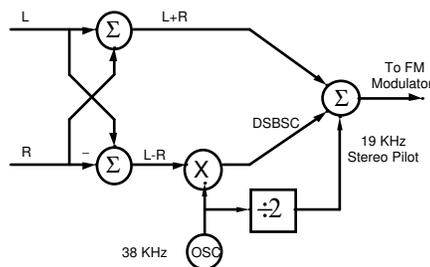
#### FM Stereo Multiplexed Baseband Spectrum





The amplitudes of the multiplexed signal components decreases as frequency increases. This prevents the carrier from deviating outside of its assigned broadcast channel.

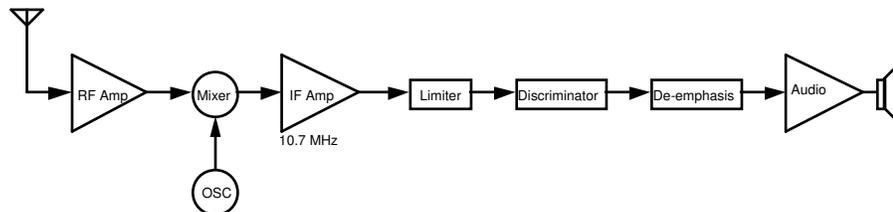
The stereo multiplex signal is quite easy to generate.



SCA is Subsidiary Communications Authority or audio subcarrier on a main station. For many years, this was the way Muzak was distributed. Today, it is used for private services such as foreign language programming and radio reading services for the blind.

### 4.5 FM Receivers

Because the S/N ratio in FM is quite high, radio transmitters tend to have relatively low output power levels. Consequently, the receivers require RF amplifiers in the front end.

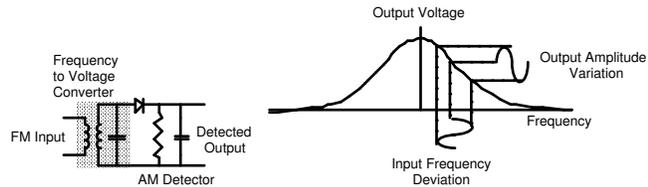


The RF and IF arrangement is the same as an AM receiver, except that the IF center frequency is 10.7 MHz. The limiter clips the IF signal, eliminating any amplitude variations.

The discriminator or FM detector, is a frequency to voltage converter. It changes frequency variations into amplitude variations. These changes in amplitude are amplified and applied to a speaker.

#### 4.5.1 FM Discriminators

The simplest FM discriminator is the slope detector.

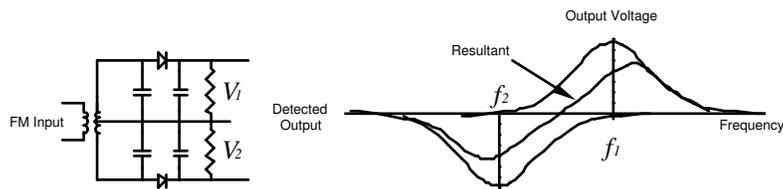


If the incoming FM signal is shifted to the slope of the tuned circuit rather than its resonant frequency, as is the case in most resonant circuits, the frequency deviation is changed to an amplitude variation.

An AM envelope is superimposed on the 10.7 MHz IF carrier at this point. By passing the signal through a standard envelope detector and low pass filter, the signal is restored to its original baseband.

The frequency and amplitude swings on this type of discriminator are somewhat limited. An improvement on this circuit is the balanced slope detector or Round-Travis detector.

##### 4.5.1.1 Balanced Slope Detector [Round-Travis]



This detector has two tuned resonant circuits. One is adjusted above the IF and the other is below followed by a pair of envelope detectors. The net result is an increased frequency and amplitude range.

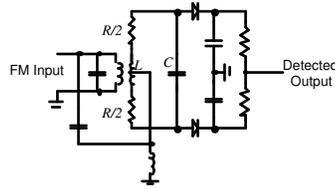
##### 4.4.1.2 Phase Detector [Foster-Seeley]

The Foster-Seeley detector converts the incoming frequency variation to an equivalent phase variation and then to an equivalent amplitude variation. This is accomplished by using the phase angle shift which occurs between the primary and secondary of a transformer tuned circuit.

It is important to recognize that the signal on the primary side gets to the secondary through two distinctly different paths:

- through the transformer via the primary winding

- bypassing the primary winding and directly into the secondary center tap



The voltage appearing on the secondary side of the transformer is given by:

$$e_s \approx e_p k \sqrt{\frac{L_p}{L_s}}$$

$e_p$  = primary voltage

$k$  = coupling coefficient

$L_p$  = primary inductance

$L_s$  = secondary inductance

This is applied to the series resonant circuit in the transformer secondary winding. The impedance of this resonant circuit is given by:

$$\begin{aligned} Z &= R + j\left(\omega L - \frac{1}{\omega C}\right) = R\left[1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)\right] \\ &= R\left[1 + j\left(\frac{\omega}{\omega_o} \frac{\omega_o L}{R} - \frac{\omega_o}{\omega} \frac{1}{\omega_o RC}\right)\right] \end{aligned}$$

Where  $\omega_o$  is the resonant frequency.

Since  $Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$ , the impedance can be written as:

$$Z = R\left[1 + j\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)Q\right]$$

Defining a new parameter:  $Y = \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}$ , we obtain:

$$Z = R[1 + jYQ]$$

The impedance phase angle is given by:  $\phi = \tan^{-1}(YQ)$ .

It is interesting to observe what happens to this angle when the input frequency varies.

Let the input frequency be of the form:  $\omega = \omega_o + \Delta\omega$ , then:

$$Y = \frac{\omega_o + \Delta\omega}{\omega_o} - \frac{\omega_o}{\omega_o + \Delta\omega} = \frac{2\omega_o\Delta\omega + \Delta\omega^2}{\omega_o^2 + \omega_o\Delta\omega}$$

But if the deviation is much smaller than the carrier:  $\Delta\omega \ll \omega_o$ , then:

$$Y \approx \frac{2\Delta\omega}{\omega_o}$$

Notice that the parameter  $Y$  varies directly with deviation. For small angle changes  $\phi \approx \tan^{-1}\phi$ . This means that the impedance phase angle varies directly with the frequency deviation. This in turn causes a variation in the currents and voltages in the secondary.

The output of the transformer consists of the vector sum of two components:

- the phase shifted signal passing through the transformer
- the unshifted signal which has bypassed the transformer

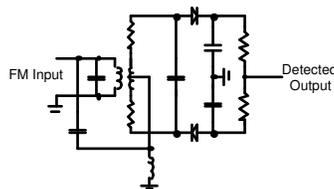
The combination of these two signals results in amplitude variations which are directly proportional to the frequency deviation. This AM signal is then detected through a standard envelope detector.

This circuit is quite sensitive however, any amplitude variations in the signal caused by varying signal strength are also detected.

#### 4.4.1.3 Ratio Detector

This circuit is a slight modification of the Foster-Seeley detector:

- one diode is reversed
- the output is taken from the combined loads

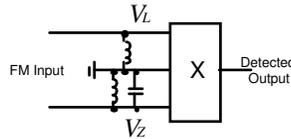


The limiting action of the detector has been improved and variations in signal strength are not as noticeable. This also implies that it is less sensitive.

4.4.1.4 Quadrature Detector

The quadrature detector also makes use of the phase shift which occurs across a tuned network, but uses a product rather than sum to generate the amplitude variations.

This technique is widely used in integrated radio circuits.



If a sinusoidal input current passes through the network, the voltage generated across the resonant network will be in phase at the resonant frequency, but that across the inductor will be displaced by 90°. When not at resonance, the phase angle in the resonant circuit will be  $\phi = \tan^{-1}(YQ)$ . Therefore the two signals can be expressed as:

$$V_L = \cos(\omega_c t) \quad V_Z = \sin(\omega_c t + \phi)$$

It is interesting to note what happens when these two signals are multiplied:

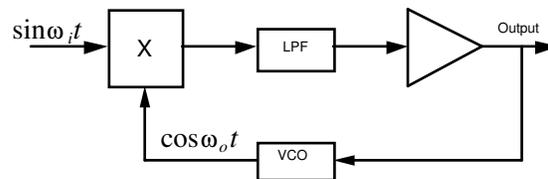
$$\begin{aligned} V_o &= \cos(\omega_c t) \sin(\omega_c t + \phi) \\ &= \frac{1}{2} \sin(2\omega_c t - \phi) + \frac{1}{2} \sin(\phi) \end{aligned}$$

The high frequency term occurs at twice the input frequency and is easily filtered off.

The low frequency term  $\sin(\phi)$  is quite interesting since for small angles, the sine of the angle is approximately equal to the angle itself  $\sin(\phi) \approx \phi$ . This means variations in phase angle cause proportional variations in amplitude. The baseband signal is then recovered in the usual way.

4.4.1.5 Phase Locked Loop

Although phase locked loops can be implemented using analog or digital circuitry, the following discussion will be limited to linear circuits since they are much easier to analyze.



The loop achieves lock in two steps. First it acquires frequency lock, and then it acquires phase lock.

If the input signal and VCO output are completely different, the output of the multiplier is given by:

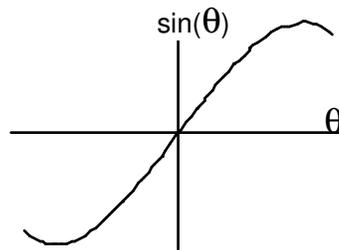
$$V_{mult} = \sin(\omega_i t) \cos(\omega_o t) = \frac{1}{2} \sin(\omega_i + \omega_o)t + \frac{1}{2} \sin(\omega_i - \omega_o)t$$

The low pass filter removes the high frequency component before passing the signal to the output amplifier. The output, which also is the input to the VCO, is of the form:

$$V_{output} = \sin(\omega_i - \omega_o)t$$

Notice that if  $\omega_i > \omega_o$ , the output is positive, but if  $\omega_i < \omega_o$ , the output is negative.

This change in polarity can be seen by observing the sine function near the origin:



This voltage can be used as an error signal to drive the VCO until  $\omega_i = \omega_o$ , and frequency lock is achieved. The two signals however, are not necessarily in phase at this point.

Once frequency lock occurs, the multiplier output becomes:

$$V_{mult} = \sin(\omega_i t) \cos(\omega_i t - \phi) = \frac{1}{2} \sin(2\omega_i - \phi)t + \frac{1}{2} \sin(\phi)t$$

Again the low pass filter removes the high frequency component and the output is of the form:

$$V_{output} = \sin(\phi)t$$

Again if  $\phi > 0$  the output is positive, but if  $\phi < 0$  the output is negative. This error signal is used to drive the VCO until  $\phi = 0$ , and phase lock is achieved. Notice that at lock, the incoming signal and the VCO output are in quadrature.

A PLL can be used after the IF amplifier in a radio to reproduce the original modulation signal. This allows the free running VCO oscillator frequency to be preset, thus making it easier to acquire lock.

## 4.6 Spread Spectrum

[Spread Spectrum Techniques for the ISM Band](#)

[A Brief Tutorial on Spread Spectrum and Packet Radio by Harris](#)

<http://www.ece.wpi.edu/courses/ee535/hwk11cd95/witek/witek.html>

<http://cas.et.tudelft.nl/~glas/thesis/>

FM is really a form of spread spectrum modulation. The term simply means that a transmitted signal is spread out over a much wider frequency band than its baseband source.

CDMA is a form of spread spectrum, where digital techniques are used to broaden out the baseband. The two most common techniques used are direct sequence and frequency hopping.

In CDMA systems, all users have access to the full transmission bandwidth. The actual spectrum used is correlated to a code which is unique to each end-user or connection. This provides a number of advantages:

- Low power spectral density
- High degree of privacy
- Soft access limit
- Reduced multi-path fading
- Minimized interference

The processing gain, the ratio between the transmission bandwidth and baseband, is used as a measure of spectral spread.

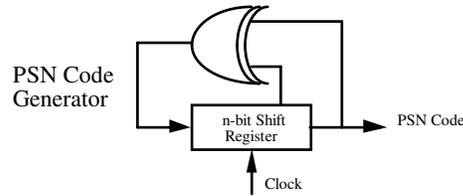
$$G_p = \frac{BW_{TX}}{BW_{BB}}$$

There are two broad categories of codes which are used in CDMA:

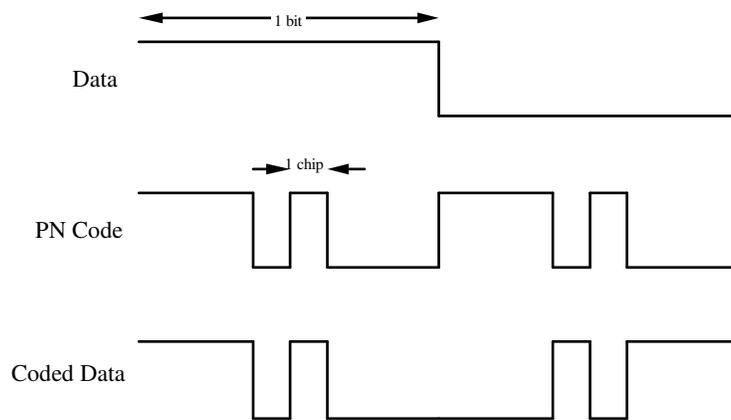
- Orthogonal– Walsh-Hammar codes
- Non-orthogonal – M-sequence, Gold, and Kasami codes

### 4.6.1 Direct Sequence

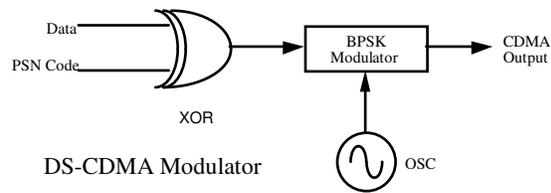
In the DS<sup>†</sup> method, a PN<sup>†</sup> sequence or code is used to artificially increase the bit rate of the baseband signal.



The code bit rate is faster than the data bit rate. The code bits, known as chips are used to modulate the input data by means of bit-inversion modulation.

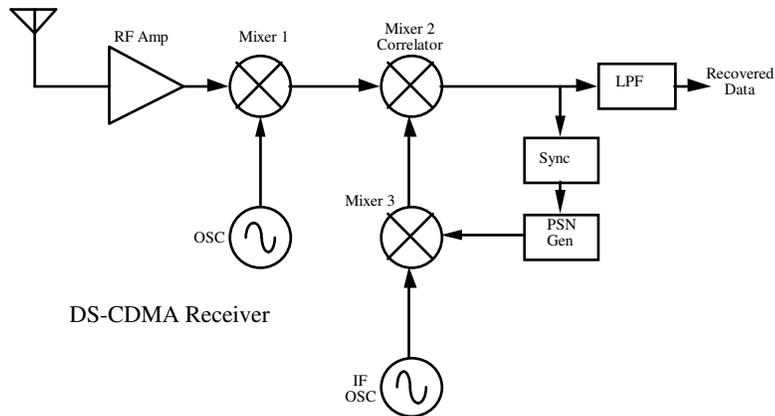


The resulting scrambled bit stream is used to BPSK or QPSK modulate an RF carrier. A direct sequence spread spectrum modulator is a relatively simple device.



The receiver is a more complex device. The decoding of the PN code is generally done in an IF mixer, also called a correlator, rather than in the baseband.

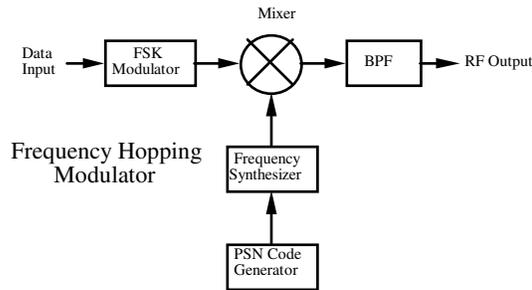
† Direct Sequence  
 † Pseudo-Noise



One of the disadvantages of DS-CDMA is the near-far problem. The receiver has difficulty maintaining lock on a weak signal if a more powerful transmitter is operating nearby. This happens if there is no correlation between the two codes being used.

### 4.6.2 Frequency Hopping

Instead of scrambling the input bit stream, the frequency-hopping scheme uses the PSN to vary the carrier frequency.



This has a tendency to reduce the near-far problem to instances when the two transmitters hop to nearby frequencies. Thus the interference is reduced.

## Assignment Questions

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### Quick Quiz

1. The maximum amount of preemphasis used in commercial FM radio is [3, 15, 17] dB at 15 KHz.
2. The modulation index in both AM and FM systems is directly proportional to the modulation signal amplitude. [True, False]
3. FM carrier deviation is directly proportional to modulation signal [amplitude, frequency, phase].
4. The [Round-Travis, Foster-Seeley] discriminator has greater sensitivity.

### Analytical Questions

1. Calculate the magnitude of the first side frequency if a 1 volt FM signal has a modulation index of 2.1.
2. Determine the approximate broadcast bandwidth when an FM station with a modulation index of 4 transmits a 10 KHz tone.
3. Calculate the magnitude of an FM carrier when the modulation index is 4.
4. What is the approximate bandwidth when an FM station with a modulation index of 3.75 transmits a 2.5 KHz tone?
5. An FM antenna delivers, to a  $75 \Omega$  antenna, a signal of:

$$v = 1000 \sin(10^9 t + 4 \sin 10^4 t)$$

Calculate:

- a) Carrier and intelligence frequencies
- b) Output power
- c) Modulation index
- d) Carrier deviation
- e) Bandwidth
- f) Amplitude of the 5th side frequency

### Composition Questions

1. Explain the need for preemphasis and why FM has a better signal to noise ratio than AM.
2. Explain how the modulation index, bandwidth, and carrier amplitude are affected by the modulation signal frequency and amplitude in an FM signal.

## For Further Research

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CDMA

<http://www.ece.nwu.edu/~phoel/cdma/>

<http://www.mot.com/CNSS/CIG/Technology/cdma.html>

<http://www.qualcomm.com/ProdTech/cdma/>

<http://www.cdg.org/>

<http://www.srs-rf.com/resource.htm>